- QUANTUM STATE TOMOGRAPHY
- CV SYSTEMS
- QUANTUM STATE TOMOGRAPHY OF CV SYSTEMS

  → - ENERGY-CONSTRAINED STATES

  → - GAUSSIAN STATES

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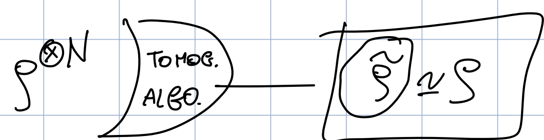


$\tilde{\rho} \simeq \rho$ $\boxed{\varepsilon > 0}$

TRACE DISTANCE : (Holevo-Helstrom theorem)

$$\frac{1}{2}\|\rho - \tilde{\rho}\|_1 := \frac{1}{2}\operatorname{Tr}|\rho - \tilde{\rho}| \; (\leq \varepsilon)$$

1) ρ or σ $\quad$ Prob $= 1/2$ $\quad \boxed{?}$

$$P_{succ}^{(BEST)} = \frac{1}{2}\left(1 + \frac{1}{2}\|\rho - \sigma\|_1\right)$$

.2) Θ

$$\left|\mathrm{Tr}[\rho\Theta] - \mathrm{Tr}[\sigma\Theta]\right| \leq \|\Theta\|_\infty \cdot \|\rho-\sigma\|_1$$

HOLDER

$$\rho^{\otimes N} \longrightarrow \boxed{\text{TOMOG. ALGO.}} \longrightarrow \boxed{\tilde{\rho} \simeq \rho} \qquad \Pr\left[\frac{1}{2}\|\tilde{\rho}-\rho\|_1 \leq \varepsilon\right] \geq 1-\delta$$

TR. DIST. ERROR

FAIL. PROB.

. $\varepsilon, \delta > 0$.

. $\rho \in S'$

SAMPLE COMPLEXITY:

. Fix $\varepsilon, \delta \in (0,1)$, S'.

. The SAMPLE COMPLEXITY $N(S', \varepsilon, \delta)$ is the minimum $N \in \mathbb{N}$ s.t. $\exists$ quantum algo s.t. given $\rho^{\otimes N}$ $(\rho \in S')$ as input it outputs $\tilde{\rho}$

$$\Pr\left[\frac{1}{2}\|\tilde{\rho}-\rho\|_1 \leq \varepsilon\right] \geq 1-\delta$$

. EX. 1 : (QUDIT)

$$S := \mathcal{D}(\mathbb{C}^d)$$

$$\boxed{N = \Theta\left(\frac{d^2}{\varepsilon^2} \cancel{\log\left(\frac{1}{\delta}\right)}\right)}$$ (ENT.) (2015)

(UNENT. = "SINGLE - COPY")

$$N = \Theta\left(\frac{d^3}{\varepsilon^2}\right)$$

BOOSTING ARGUMENT

. EX2 (PURE QUDIT)

$$S := \{ |\psi\rangle \in \mathbb{C}^d : \langle\psi|\psi\rangle = 1 \}$$

$$\boxed{N = \Theta\left(\frac{d}{\varepsilon^2}\right)}$$

CV SYSTEMS :

1 MODE := 1 QUDIT WITH DIMENSION $= \infty$

$$\mathcal{H} = \mathrm{Span}\{|0\rangle, |1\rangle, |2\rangle, \dots, |d\rangle, |d+1\rangle, \dots\}$$

FOCK STATES

$$\boxed{\mathcal{H}^{\otimes n}}$$

$$\boxed{N = \infty} \qquad (\dim = \infty)$$

ONE MODE:

$$\hat{N} := \sum_{k=0}^{\infty} k\,|k\rangle\langle k| \qquad (= a^\dagger a)$$

$$\mathrm{Tr}[\rho \hat{N}] \leq E$$

$$\mathcal{S}_E := \{\rho \in \mathcal{H} \ : \ \mathrm{Tr}[\rho \hat{N}] \leq E\}$$

$$\boxed{\begin{array}{l} \text{• GENTLE MEAS. LEMMA:} \\ \Pi, \rho \quad \rightsquigarrow \quad \frac{1}{2}\left\| \rho - \frac{\Pi \rho \Pi}{\mathrm{Tr}[\Pi \rho]} \right\|_1 \leq \sqrt{\mathrm{Tr}[(1-\Pi)\rho]} \end{array}}$$

$(\Pi, 1-\Pi)$

$$\Pi_M := \sum_{k=0}^{M} |k\rangle\langle k|$$

$$\mathrm{Tr}[\rho \hat{N}] \le E$$

$$= \boxed{\sum_{k=0}^{\infty} k \langle k|\rho|k\rangle \le E} \quad (*)$$

$$\frac{1}{2}\left\| \rho - \frac{\Pi_M \rho \Pi_M}{\mathrm{Tr}[\Pi_M \rho]} \right\|_1 \le \sqrt{\mathrm{Tr}[(1-\Pi_M)\rho]}$$

$$= \sqrt{\sum_{k=M+1}^{\infty} \langle k|\rho|k\rangle}$$

$$\le \sqrt{\sum_{k=M+1}^{\infty} \frac{k}{M} \langle k|\rho|k\rangle}$$

$$\le \sqrt{\sum_{k=0}^{\infty} \frac{k}{M} \langle k|\rho|k\rangle}$$

$$= \sqrt{\frac{\mathrm{Tr}[\rho \hat{N}]}{M}}$$

$$\le \sqrt{\frac{E}{M}} \overset{!}{\le} \varepsilon$$

$$\hookrightarrow \quad M \ge \frac{E}{\varepsilon^2}$$

$$M := \left\lceil \frac{E}{\varepsilon^2} \right\rceil$$

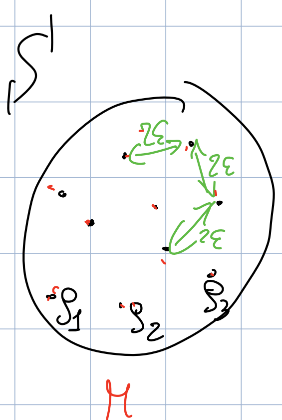
$(\Pi_M, \mathbb{1} - \Pi_M)$ $\quad \rho \quad M$

$$\rho \rightsquigarrow \frac{\Pi_M \rho \Pi_M}{\mathrm{Tr}[\Pi_M \rho]} \in D(\mathbb{C}^M)$$

$$N = O\left(\frac{M^2}{\varepsilon^2}\right) = O\left(\frac{E^2}{\varepsilon^6}\right) \quad \text{(MIXED)}$$

$$N = O\left(\frac{M}{\varepsilon}\right) = O\left(\frac{E}{\varepsilon^4}\right) \quad \text{(PURE)}$$

LOWER BOUNDS:



ALICE picks $i \in \{1, \dots, M\}$ $\qquad\qquad$ BOB

$\rho_i^{\otimes N} \longrightarrow$ TOM, ALGO $\rightsquigarrow i$

M messages

$\log_2 M$ bits

HOLEVO THEOREM

QUDIT $\dim = d$

$$\log_2 M \leq \log_2 \dim \mathcal{H}_d^{\otimes N}$$

$$= \log_2 d^N$$

$$= N \log_2 d$$

$$\boxed{N \geq \frac{\log_2 M}{\log_2 d}}$$

n modes :

$$\hat{N}_{Tot} = \sum_{j=1}^{n} \hat{N}_j$$

$\mathrm{Tr}[\rho \hat{N}_{TOT}] \leq n E$

PURE :

$N = \tilde{\Theta}\left(\frac{E^n}{\varepsilon^{2n}} \right)$

Pure

FINITE :

$\frac{1}{\varepsilon^2}$

$N \mapsto 4N$

CV :

$\frac{1}{\varepsilon^{2n}}$

$N \mapsto 4^n N$

$\varepsilon \mapsto \frac{\varepsilon}{2}$

GAUSSIAN STATES :

n modes

ρ GAUSS $\longleftrightarrow$ $m \in \mathbb{R}^{2n}$ $(2n)$

$V \in \mathbb{R}^{2n \times 2n}$ $(4n^2)$

HETERODYNE

$$\rho^{\otimes N} \rightsquigarrow \Pr\left[\| \tilde{V} - V \| \leq \varepsilon \right] \geq 1 - \delta$$

$$\frac{1}{2} \| \tilde{\rho} - \rho \|_1 \leq \varepsilon$$

$$\rho_{m,V}^{\otimes N} \quad \begin{matrix} \tilde{m} \\ \tilde{V} \end{matrix} \rightsquigarrow \rho_{\tilde{m},\tilde{V}} \underset{\varepsilon}{\approx} \rho_{m,V}$$

$m = 0$

$$\underbrace{\frac{1}{2} \| \rho_V - \rho_W \|_1}_{\leq 0} \leq \frac{1+\sqrt{3}}{8} \operatorname{Tr}\left[\left(\frac{V^{-1} + W^{-1}}{2} \right) \underbrace{|V - W|}_{\leq 0} \right]$$

HETERODYNE $\longrightarrow$ $\boxed{N = O\left(\frac{n^3}{\varepsilon^2} E^2 \right)}$

$\left(\operatorname{Tr}[\rho \hat{N}_{TOT}] \leq nE \right)$

ADAPTIVE - UNSQUEEZING PROTOCOL $\rightsquigarrow$ $\boxed{N = O\left(\frac{n^3}{\varepsilon^2} \log\log E\right)}$

$E = 10^{300}$ $\rightsquigarrow$ $\log\log E \simeq 10$

$$d_{eff} = \frac{E}{\varepsilon^2}$$

$\eta_{eff} := \ldots$

$$O\left(\frac{d_{eff}\,\eta_{eff}}{\varepsilon^2}\right)$$

$$NG \xleftarrow[\text{T-DOPED}]{} G$$

NON-EFFICIENT

EFFICIENT