NLTS Hamiltonians: Two Perspectives

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Outline: Part I

This part covers the paper **"NLTS Hamiltonians from Good Quantum Codes"**.

Quantum PCP Conjecture

The Quantum PCP Conjecture asks whether approximating the ground state energy of local quantum Hamiltonians is QMA-hard.

This is a quantum analog of the classical PCP theorem, which revolutionized hardness of approximation.
Proving NLTS is considered a crucial step toward resolving the Quantum PCP Conjecture.

NLTS Conjecture

No Low-Energy Trivial States (NLTS) posits that there exists a family of local Hamiltonians such that all low-energy states require high-depth quantum circuits to prepare.

In other words, the structure of these Hamiltonians prevents trivial (shallow) quantum circuits from approximating the ground state.

Trivial States and Circuit Depth

A state is **trivial** if it can be generated by a quantum circuit of polylogarithmic depth.

The challenge is to prove that any state close to the ground energy cannot be generated by such shallow circuits — that is, it must be complex.

Quantum LDPC Codes

LDPC stands for Low-Density Parity-Check codes. These are quantum error-correcting codes with:

- Local stabilizer generators
- Constant rate and linear distance (”good” codes)

Good LDPC codes form the foundation for constructing NLTS Hamiltonians.

Geometric Expansion and Local Testability

A code is **locally testable** if checking a few bits suffices to infer global correctness.

Codes with strong expansion properties ensure that local errors affect many checks — essential for robustness.

Main Theorem

There exists a family of local Hamiltonians $H_n = \sum h_i$ such that:

- Each h_i acts on $O(1)$ qubits
- Every low-energy state requires circuit depth $\Omega(\log n)$

Circuit Lower Bounds

The proof leverages:

- **Uncertainty principle**: shallow circuits cannot localize all observables
- **Testability**: measurement errors cannot be hidden locally
- **Expansion**: ensures errors spread across the system

Uncertainty Principle

Quantum Version

$$\Delta A^2 + \Delta B^2 \geq \text{const.} \quad \text{for non-commuting A, B}$$

Shallow circuits fail to suppress variance in non-commuting measurements, leading to spread in measurement outcomes.

Final Implications

- This is one of the first constructions proving NLTS using good LDPC codes.
- Establishes a bridge between quantum error correction and circuit complexity.
- Paves a path toward resolving the Quantum PCP Conjecture.

Outline: Part II

This part covers the paper **"NLTS and SoS Lower Bounds from LDPC Codes"**.

1. Background and Motivation
2. Quantum Codes and Circuit Complexity
3. Main Result and Techniques
4. Background and Goals
5. Construction and Analysis
6. Conclusion and Impact

Sum-of-Squares (SoS) Overview

The Sum-of-Squares hierarchy is a powerful framework for approximating solutions to optimization problems.

It generalizes semidefinite programming and is used to reason about hardness of approximation in both classical and quantum settings.

SoS and Quantum Hamiltonians

Applying SoS to quantum Hamiltonians asks:

- Can low-degree SoS relaxations approximate ground state energy?
- If not, this implies computational hardness of quantum systems even for approximation.

LDPC Codes and Hamiltonians

As in the first paper, LDPC codes form the base.

- Stabilizer generators form local constraints
- Projectors derived from code checks

These build the Hamiltonian to be analyzed under SoS.

Main Theorem (SoS Hardness)

Any SoS algorithm of degree $d = o(n^{\epsilon})$ cannot approximate the ground energy of H within small error.
This is an *unconditional* lower bound — SoS provably fails for these systems.

Pseudoexpectation Construction

A pseudoexpectation is a fake solution that satisfies all low-degree SoS constraints:

- Mimics valid moment constraints
- Suggests low energy falsely
- SoS accepts it as valid, missing true complexity

Explicit Code Construction

The authors construct explicit LDPC codes using Ramanujan complexes and other combinatorial objects.

These constructions are:

- Efficiently verifiable
- Have strong expansion and sparsity
- Low rate but robust structure

Consequences and Open Questions

- Shows SoS fails for low-rate LDPC-based Hamiltonians
- Reinforces separation between classical relaxations and quantum complexity
- Future: Can this be extended to higher-rate codes or other proof systems?

Summary of Contributions

- Paper 1 constructs NLTS Hamiltonians using good LDPC codes
- Paper 2 proves SoS lower bounds using LDPC Hamiltonians
- Both advance the landscape of quantum PCP and complexity

Thank you!

Questions?